

Total No. of Questions - 24

Regd.

Total No. of Printed Pages - 3

No.

Part - III

MATHEMATICS, Paper - I (A)

(Algebra, Vector Algebra and Trigonometry)

(English Version)

Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of three sections A, B and C.

SECTION A

I. Very short answer type questions.

10 × 2 = 20

i) Answer **all** questions.ii) Each question carries **two** marks.1. If $f = \{(1, 2), (2, -3), (3, -1)\}$, then find :i) $2f$ ii) $2 + f$ 2. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 25}$.3. Let $\vec{a} = 2\vec{i} + 4\vec{j} - 5\vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{c} = \vec{j} + 2\vec{k}$. Find the unit vector in the opposite direction of $\vec{a} + \vec{b} + \vec{c}$.4. Find the vector equation of the plane which passes through the points $3\vec{i} + 4\vec{j} + 5\vec{k}$, $5\vec{i} + 2\vec{j} + 3\vec{k}$ and parallel to the vector $3\vec{i} - 2\vec{j} + \vec{k}$.5. If $\vec{a} = (4, 3, 5)$ is the center of a sphere which passes through the point $(-1, -1, 2)$, then find the cartesian equation of the sphere.

6. Find the period of the function defined by

$$f(x) = \sin(x + 2x + \dots + nx) \text{ for all } x \in R \text{ and } n \in Z^+.$$

7. If $3\sin\theta + 4\cos\theta = 5$, then find the value of $4\sin\theta - 3\cos\theta$.
8. If $\sinh x = \frac{3}{4}$, find $\cosh(2x)$ and $\sinh(2x)$.
9. Show that in a $\triangle ABC$, $b \cdot \cos^2 \frac{C}{2} + c \cdot \cos^2 \frac{B}{2} = s$.
10. If the amplitude of $(z - 1)$ is $\frac{\pi}{2}$, then find the locus of z where $z = x + iy$.

SECTION B

II. Short answer type questions.

5 × 4 = 20

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.
11. If $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors, then show that $\vec{a} - 3\vec{b} + 2\vec{c}$, $2\vec{a} - 4\vec{b} - \vec{c}$ and $3\vec{a} + 2\vec{b} - \vec{c}$ are linearly independent.
12. If $0 \leq \alpha, \beta \leq \pi$, then show that $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$ by the vector method.
13. If A is not an integral multiple of π , prove that

$$\cos A \cdot \cos 2A \cdot \cos 4A \cdot \cos 8A = \frac{\sin 16A}{16 \sin A}.$$
14. Solve the equation $\cot^2 x - (\sqrt{3} + 1)\cot x + \sqrt{3} = 0$ where $\left(0 < x < \frac{\pi}{2}\right)$.
15. Find the value of $\tan\left(\cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3}\right)$.
16. If $\sin\theta = \frac{a}{b+c}$, then show that $\cos\theta = \frac{2\sqrt{bc}}{b+c} \cos \frac{A}{2}$.
17. Show that $16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$.

SECTION C

III. Long answer type questions.

5 × 7 = 35

- i) Attempt **any five** questions.
- ii) Each question carries **seven** marks.

18. Let $f : A \rightarrow B$ be a bijection, then show that $f \circ f^{-1} = I_B$ and $f^{-1} \circ f = I_A$.

19. Using mathematical induction, prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots \text{ upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12} \quad \forall n \in \mathbb{N}.$$

20. If $A = (1, -2, -1)$, $B = (4, 0, -3)$, $C = (1, 2, -1)$, $D = (2, -4, -5)$, then find the distance between the lines AB and CD .

21. If $A + B + C = 180^\circ$, then prove that

$$\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 \left(1 + \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right).$$

22. In a $\triangle ABC$, show that $r + r_1 + r_2 + r_3 = 4R \cos C$.

23. On a tower AB of height h , there is a flag staff BC . At a point 'd' meters away from the foot of the tower, AB and BC are making equal angles.

Show that the height of the flag staff is $h \left(\frac{d^2 + h^2}{d^2 - h^2} \right)$ meters.

24. If n is an integer and $z = cis \theta$, $\left(\theta \neq (2n+1)\frac{\pi}{2} \right)$, then show that

$$\frac{z^{2n} - 1}{z^{2n} + 1} = i \tan n\theta.$$